# ACALCULIA (HENSCHEN)

## A CLINICAL STUDY

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Acalculia is an inability to perform simple arithmetic operations; it may occur either as a concomitant symptom within an aphasic syndrome or as the main or even the only symptom of a focal cerebral lesion. As a concomitant symptom it is probably associated with many of the common aphasias. Henschen [1] was able to collect from the literature no less than 260 cases in which acalculia was mentioned as an accompanying condition. That in many more cases it had been present but was overlooked appears to be a safe assumption.

Cases in which acalculia was the outstanding or exclusive symptom of a cerebral lesion have been described by Lewandowsky and Stadelmann,[2] Peritz,[3] Sittig,[4] Berger,[5] Gerstmann,[6] Herrmann [7] and Lange.[8] The views expressed in these publications on the nature of the condition are sufficiently controversial and obscure to warrant the addition of a new case to the list of those already published.

### REPORT OF A CASE

*Clinical History.*—A white man, aged 44, an assistant superintendent of a firm of boiler and water tank engineers, was admitted to the Edward Hines, Jr.,

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1. Henschen, S. E.: Klinische und anatomische Beiträge zur Pathologie des Gehirns, Stockholm, 1920, pt. 5.

2. Lewandowsky, M., and Stadelmann, E.: Ueber einen bemerkenswerten Fall von Hirnblutung und über Rechenstörungen bei Herderkrankung des Gehirns, J. f. Psychol. u. Neurol. **11**:249, 1908.

3. Peritz, G.: Zur Psychopathologie des Rechnens, Deutsche Ztschr. f. Nervenh. **61**:234, 1918.

4. Sittig, O.: Störungen des Ziffernschreibens und Rechnens bei einem Hirnverletzten, Monatschr. f. Psychiat. u. Neurol. **49**:299, 1921.

5. Berger, H.: Ueber Rechenstörungen bei Herderkrankungen des Grosshirns, Arch. f. Psychiat. **78**:238, 1926.

6. Gerstmann, J.: Fingeragnosie und isolierte Agraphie, ein neues Syndrom, Ztschr. f. d. ges. Neurol. u. Psychiat. **108**:152, 1927.

7. Herrmann, G.: Beiträge zur Lehre von den Störungen des Rechnens bei Herderkrankungen des Okzipitallappens (Akalkulie Henschen), Monatschr. f. Psychiat. u. Neurol. **70**:193, 1928.

8. Lange, J.: Fingeragnosie und Agraphie, Monatschr. f. Psychiat. u. Neurol. **76**:129, 1930.

Memorial Hospital on Oct. 3, 1930, with the tentative diagnosis of a "traumatic psychosis." The family history was without significance. The patient had had no previous illnesses. He had attended school to the age of 11, when he went to work, but he had attended night school until the age of 16. He had been employed for a number of years as an assistant superintendent by a responsible firm, and at times had fifty men working under him. His work required that he estimate the cost of jobs and keep track of time and the wages of the men under him. His employers speak of his work in the highest terms. He served in the Navy during the World War, and had since been employed steadily.

On Feb. 10, 1930, he drove into his garage, and, shortly afterward, was found unconscious on the floor. The neighbors who found him reconstructed the story as follows: He had driven into the garage and raised the hood of the car to make an inspection. The door of the garage blew closed. He slipped and in doing so struck the accelerator, set the motor racing and became asphyxiated. He was taken to the Auburn Park Hospital. A blood count, made on February 13, showed 4,880,000 red cells, 17,200 white cells and 95 per cent hemoglobin. No other laboratory tests were made. On February 13 and 14, "his condition was much improved although he was unable to see" This is not quite in accord with the condition reported on February 14.

On February 14, he was admitted to the Wesley Memorial Hospital, where he regained consciousness after two days. "However, he suffered from total loss of vision for the following three weeks," and had frequent crying spells. On March 11, he returned home, and it was noted that when he started to talk on a subject he harped on it for a long time. He had lost his memory "for practically everything."

When admitted to the Edward Hines Hospital it was noted that he was unable to dress himself and would put the left leg into the right leg of his trousers. He was equally awkward in attempts to put on his coat or shirt, and had to be assisted in these acts. He complained that everything seemed "left-sided" with him. For example, his bed sheet was rumpled and needed straightening on the right edge of the bed; no matter how often he attempted to pull it to the right, he always pulled to the left. In writing numbers he reversed the position of the digits. Memory for remote events was said not to be disturbed. In performance tests he was apraxic. He could tell the position of the hands of a clock but could not indicate the time. He was able to recognize tools and their use, but was unable to handle them; e.g., he knew a hammer and a nail, but he could not drive a nail.

A physical examination, made on October 3, gave completely negative results as to the cranial nerves, reflexes, motility, sensibility, coordination, gait and the Romberg test. Vision was 20/30 in the right eye and 20/20 in the left eye. The fundi were normal. Repeated tests demonstrated that no hemianopia was present. The range of hearing was 12 feet (365.7 cm.) with the right ear and 10 feet (304.8 cm.) with the left ear for the conversational voice; for the whispered voice, it was 5 feet (152.4 cm.) with the right ear and 3 feet (91.4 cm.) with the left ear.

Laboratory Tests: The urine was normal; the Wassermann and Kahn reactions of the blood were negative. A blood count showed: 4,000,000 red cells and 10,000 white cells, of which 68 per cent were polymorphonuclears, 20 per cent small mononuclears, 8 per cent large mononuclears, 2 per cent eosinophils and 2 per cent transitional cells; the hemoglobin was 80 per cent; the color index was 1. The spinal fluid showed a pressure of 30; the globulin and cell content were normal; the Wassermann reaction was negative; the Lange gold curve was 00011100000.

A diagnosis of encephalitis due to carbon monoxide poisoning was made.

*Examination for Aphasia.*—On October 29, a special examination was made for aphasic symptoms by the method described by the junior author.[9] It was found that the patient had some slight difficulty at times in finding words, but he was able to name objects shown to him, to indicate objects named verbally or in print and to designate the use of objects. He was able to repeat sense and nonsense words and sentences of considerable length and complexity. Reading was good for both sense and nonsense material, and for letters, syllables, words, sentences and paragraphs. The only mistakes that he made in reading referred to the pairs of letters p and q and b and d, which were often confounded when they were offered as isolated letters. In their context within a word, sentence or paragraph, these letters were never exchanged for each other.

In tests for eupraxis, the patient was well able to imitate even complex movements and to carry out complex orders of various kinds, but he was unable to carry out a total act the parts of which involved a continuous change in spatial arrangement, such as tying a knot or wrapping a book and tying a string crosswise around it.

The main defects lay in the sphere of arithmetic and writing. In these he proved well nigh helpless. Writing, whether spontaneous, dictated or copied, was reduced to a hardly legible scribble. So far as it could be deciphered, it suggested that the patient wrote mainly in response to auditory stimuli. The words were written as they were spoken, phonetically: Lincoln was spelled Lincelen, and captain, capten. In calculation he proved completely defective. Even the simplest operation, like 5 plus 6, could not be done.

Since acalculia and agraphia have been shown by Gerstmann [6] to be frequently associated with chromagnosia (inability to recognize colors) and with finger agnosia (inability to manipulate, recognize, indicate and name the individual fingers), color perception and finger manipulation were carefully tested. The patient showed great facility in sorting and naming colors. He was even able to arrange the various color shades of the hemoglobin scale after some initial mistakes. He manipulated his fingers rather clumsily, but was able to name, indicate and show them correctly on request after some initial mistakes and subsequent instruction. However, he volunteered the information: "It took me two months before I could distinguish my fingers." From this one may conclude that a finger agnosia had been present in the beginning of the disease.

It was observed on several occasions that the patient apparently failed to perceive objects lying in the right half of the visual field. For example, in building words from cards bearing single letters he would fail to use any that happened to lie to the right of the central vision. As already recorded, he also was unable to straighten the sheet on the right side of his bed. This is of interest in that Lewandowsky and Stadelmann,[2] Peritz,[3] Gerstmann,[6] Herrmann [7] and Lange [8] reported the presence of hemianopia in association with acalculia. Repeated perimetric studies, using stimuli of various sizes, failed to reveal any hemianopia or scotoma. Tests for the recognition of objects, their size, number or geometric pattern when located in the peripheral fields of vision failed to reveal any differences between the two lateral halves of the patient's fields of vision or from the results with normal controls. Hence, we are unable to explain the deficiencies noted in the vision of our patient.

9. Low, A. A.: A Case of Agrammatism in the English Language, Arch. Neurol. & Psychiat. **25**:556 (March) 1931.

On the basis of this preliminary examination the disturbance was summed up as an acalculia plus agraphia, with some apraxic but no gross aphasic symptoms. More detailed studies were then undertaken and will be described.

CALCULATION

1. *"Pure" Calculation.*—(*a*) Description of Mistakes: In mental figuring the patient was unable, even after six months of daily instruction, to make the simplest subtractions and divisions. Once only did he say that 10 minus 2 equals 8. This isolated success must be rated as random. In divisions he never reached a correct solution even in a chance answer; when asked, for example, how much is 6 divided by 2, he would think and strain and finally say: "I don't know that." In additions, he had preserved the faculty of adding 2 digits the sum of which was less than 10, as in $4 + 3 = 7$, and of adding a digit to 10 or 20, as in $10 + 8 = 18$; $20 + 7 = 27$. In all other additions he made constant mistakes, as in $6 + 5 = 12$; $11 + 2 = 12$; $14 + 6 = 32$. Simultaneous addition of three digits was practically always missed: $1 + 3 + 4 = 6$; $4 + 5 + 6 = 14$. In multiplications alone did he show some moderate progress toward the fourth month of intensive daily training. At that time he succeeded in managing the simple multiplication table and was able to execute, with only minimal mistakes, operations like $7 \times 2 = 14$; $6 \times 12 = 72$.

After writing down the problems dictated (here called figuring to dictation), he was completely defective and incapable of learning the simplest subtraction and division. In additions, his capacity was limited to primitive operations, like $3 + 6 = 9$; $10 + 6 = 16$. Anything more complex was bungled: $14 + 6 = 21$; $17 + 4 = 23$. In multiplications, he showed from the onset some proficiency in such simple performances as $2 \times 5 = 10$; $6 \times 4 = 24$; but the number of mistakes far outweighed the average of correct solutions. However, toward the end of the period of observation he made, under training, similar progress as in mental figuring.

When the problem, instead of being dictated, was written on a slip of paper and exposed for from five to fifteen seconds, and the patient was asked to figure the problem out in writing, the result showed the same types and approximately the same proportion of mistakes as in mental figuring and in figuring to dictation.

When an addition was carried out on paper, in the customary columnar arrangement, the vast majority of the results were faulty.

| | | |
|---|---|---|
| 5 | 12 | 123 |
| 6 | 15 | 258 |
| — | — | — |
| 12 | 27 | 911 |

However, considerable progress was noted in written out calculations in the fourth month of training. The improvement extended even to subtraction, but not to division.

*Comment.*—One essential point in which mental figuring differs from dictated, exposed and completely written out calculations is the entire absence of visual facilitation in the former operation as against the proportionately increasing degree of visual facilitation in the last three operations. Since the introduction and graded accentuation of the visual factor in the last three operations gave no improvement in the results, the inference was that the fundamental defect could not be

ascribed solely to the patient's being of either a predominantly "visual" or a predominantly "auditory" type. However, some sort of visual element, though not in the nature of facilitation, seemed to be operative in some mistakes which will be cited later.

(*b*) Analysis of Mistakes: When the patient said or wrote $36 + 5 = 33$ and, on being checked, "corrected" the answer to 30, the response was plainly a random solution and left no room for an interpretation on the basis of an underlying mechanism. The majority of mistakes, both in mental figuring and in written operations, however, revealed well defined mechanisms. They were: (1) substitution of one operation for another, as described by Berger;[5] (2) substitution of counting for calculation; (3) recapitulation of a digit, and (4) reversal of digits.

Illustrations of these mechanisms are: (1) A substitution of operations took place in $2 + 4 = 8$; and $2 + 8 = 16$. In these instances the patient, when asked to do an addition, performed a correct multiplication instead. An analogous substitution was $2 \times 6 = 8$; $2 \times 5 = 7$. Addition was here substituted for multiplication.

A type of substitution that was rather unexpected was $8 + 5 = 3$; $7 + 2 = 5$; $6 + 5 = 1$; $9 + 3 = 6$, when subtraction was substituted for addition. This observation was in striking contrast with the fact that the patient failed completely, on request, to perform even the simplest subtraction. The conclusion was drawn that the patient was able at times to perform a simple subtraction *spontaneously* but never on request, i. e., as a *"task."*

(2) Substitution of counting for calculation occurred in $5 + 7 = 8$; $16 + 6 = 17$, and $14 + 13 = 15$. In the example $5 + 7 = 8$, the patient counted from 7 to 8. In $16 + 6 = 17$, he counted from 16 to 17. In $14 + 13 = 15$, the counting process needs no explanation.

(3) A recapitulation of a digit was effected in $5 \times 4 = 24$. The 4 of the multiplicator was recapitulated in the product. Similar recapitulations were $32 + 5 = 45$, and $6 \times 7 = 47$.

(4) Examples of reversals are: $23 + 6 = 32$; $13 + 6 = 31$. The digits of the augmend were reversed in the sum.

No difference was noted, either in the type or in the proportion of mistakes, between mental figuring and written calculations. In the recapitulations and reversals the previously mentioned visual element came into operation.

(*c*) Time Factor: In mental figuring it was possible only to measure the time interval that elapsed between the offering of the task and the rendering of the solution, i. e., for the total process. For $7 + 5 = 13$, the total lapse was eighteen seconds. In dictated, exposed and completely written out calculations, a fractioned measuring was applied, separate times being registered for the act of writing down the task, for the intervening act of deliberation and for the final act of writing the result. Thus, for the problem $8 + 5 = 14$, the respective times were ten seconds for writing $8 + 5$; twenty-one seconds for deliberation, and six seconds for writing 14. Attempts to find out whether correct performances required more time than mistakes or vice versa proved futile. There was no regularity in the time element with regard to mistakes and correct results. In those operations only in which the patient had from the outset retained a limited proficiency, as in $3 + 4 = 7$ and $10 + 2 = 12$, was there a regular reduction of the time interval, the various times for the solution ranging between two and ten seconds, with an average of about six seconds per solution. Toward the end of the period of observation, such primitive additive problems still required from two to ten seconds, while simple multipli-

cations were regularly solved instantaneously, with hardly a measurable time interval. This conspicuous elimination of the interval proved that the multiplication table had actually been "learned," i. e., had become a mechanically mastered process. The patient had no longer to calculate and deliberate, but could reproduce the learned result from memory. He had reached the stage, generally aimed at in school training in arithmetic, when simple operations become automatic. That this result was achieved in multiplication only is apt to throw light on a basic difference between this operation and the others.

(*d*) General Approach to Mathematical Problems: After the patient had either solved or failed in a given task, he was asked to repeat the problem. In this manner his capacity for retaining a problem in mind (immediate retention) was tested:

26 + 5 = 32. (What did I ask?) Answer: 16 + 30 + 5.
36 + 5 = 33. (What did I ask?) Answer: 25 + 6 —.
1 + 3 + 4 = 6. (What did I ask?) Answer: 4 + 2 is 6.
2 + 5 = 10. (What did I ask?) Answer: 2 × 5 —.
2 × 5 = 7. (What did I ask?) Answer: 2 and 7.

*Comment.*—If in the problem 26 + 5 the numbers 26 and 5 are called the components and the word "plus" the sign, the patient was obviously not able to retain either components or signs. But the possibility was that it was not so much a matter of retention as one of recognition and understanding. In addition, if it was a matter of retention, it is possible that while the patient was unable to retain components and signs within the context of a mathematical problem, he might be able to retain them if they were offered outside such a context. To examine these possibilities the following tests were devised:

Manipulating components and signs outside mathematical contexts. The patient was given instructions about the meaning of the signs "plus," "minus," "times" and "divided by," and was asked to write the symbols (+ — × ÷) to dictation. He performed well in many repetitions. When he was again offered mathematical problems, he immediately confused the signs in the context of the task. He was then asked to write down series of 3, 4 and 5 numbers which were dictated simultaneously: 3, 6, 10 were written correctly in seven seconds; 4, 5, 9, 10 in five seconds; 5, 7, 14, 19 were changed to 5, 7, 19, 14 in sixteen seconds; 2, 5, 7, 9, 10 were written correctly in seven seconds; 11, 13, 17, 20 were rendered as 11, 31 (instead of 13), 12, 30. When the patient was directed to write similar series of simple numbers that were exposed in writing for a brief period, the results were practically the same.

*Comment.*—The tests showed that the patient had a fair span for immediate retention of both components and signs outside the context of a mathematical problem. If he had difficulty in retaining them within the context of the problem it was obviously due to some complexity inherent in the "context."

Additional information on the patient's approach to mathematical problems was obtained from an examination of his written performances. Here again, components and signs were not retained. In addition, it was observed that when he was asked to write 4 + 6 he invariably wrote 6 + 4; 3 + 6 was changed to 6

+ 3; 5 + 7 to 7 + 5; 5 + 6 to 6 + 5, etc. In other words, the higher digit was always placed as augmend, the lower digit as addend. The reverse change never took place.

Browne,[10] in an experimental study of fundamental arithmetical functions, found that college students had considerably more difficulty with those addition columns in which the augmend was smaller than the addend and proportionately less difficulty with columns in which the augmend was larger. The behavior of the patient, therefore, seems to indicate that by starting out with the larger number he tended to facilitate his task.

*Summary of the Span for "Pure" Calculation.*—The inventory taken at the beginning of the study showed some residual proficiency in the simplest additions and multiplications, but a complete deficit in subtractions and divisions. Under training, mental figuring improved in multiplication only. Written calculations showed a marked progress in both addition and multiplication, some slight advance in subtraction and a complete standstill in division. The mechanisms underlying the mistakes were: substitutions of operations, substitutions of counting for calculation, recapitulations and reversals. The time factor showed no regularity in its relation to correct and wrong solutions. Immediate retention for mathematical problems was very poor, both for the components and for the signs of a problem. However, retention was fair for both of these elements outside a mathematical context. In additions, the larger number was always placed in the position of the augmend.

2. *Application of Arithmetical Thinking to Experience.*[11]—(*a*) **Mental Figuring** with Objects: Addition. The patient was asked: A man has 5 automobiles and 3 trucks. How many vehicles? Answer: "7 trucks." (What did I ask?) Answer: "5 trucks and 3 automobiles are 7—8." In repeated attempts he was unable to hold apart automobile, trucks and vehicles. He constantly repeated: 5 trucks and 3 automobiles or: 3 trucks and 3 vehicles, etc. Three men and 6 women were correctly added together as 9 persons; 6 shirts and 4 suits were correctly given as 10 pieces of wearing apparel. But when the patient was asked to add together 2 groups of objects the sum of which exceeded 10, he failed in most questions: 12 apple trees and 5 cherry trees were given as 7 trees altogether (spontaneous subtraction); then it was "corrected" to 15 (recapitulation of 5). Addition of 3 groups of objects, like the computation of the total population of a village that has 20 men, 15 women and 40 children, was not even attempted. Addition of coins gave similar results: 2 dimes and 1 nickel were promptly computed into 25 cents; but 2 dimes, 1 nickel and 3 cents were added into 21 cents. Adding of time elements resulted in complete failure. He was told: It is now 8 o'clock. What will it be in three hours? Answer: "5 o'clock" (spontaneous subtraction). The addition of spatial entities was also completely beyond his span. Question: A man is on the fourth floor and goes up 5 stories. Where will he land? He was unable to answer. Likewise he was unable to repeat the problem that had been asked.

10. Browne, C. E.: The Psychology of the Simple Arithmetical Processes, Am. J. Psychol. **17**:1 (Jan.) 1906.

11. A few examples only of the tests contained in the protocols are here quoted.

*Comment.*—In adding 5 automobiles and 3 trucks, the patient had to deal (1) with numerical components (5 and 3), (2) with objective components (automobiles and trucks) and (3) with the sign "and." The introduction of the objective component aggravated the task, and the performances were generally far inferior to those in "pure" calculation. Questions pertaining to time elements and spatial entities were never correctly answered. Recapitulations and substitutions of operations were frequent. Of 19 questions, 7 were answered correctly.

Multiplication. The results were decidedly better than in addition. Of 24 questions, 12 were answered correctly. The type of questions asked was: A sheep has 4 legs. How many legs have 15 sheep? How many quarters are there in 3 dollars? How much do 2 cigars cost if one costs 15 cents? Questions pertaining to time and space elements yielded again a total failure.

Subtraction. Four correct answers were rendered to a total of 17 questions. The type of questions was: 20 boys are in a class; 3 leave the room. How many are left? A man has 2 fingers of his hand shot off. How many fingers are left?

Division. Of 10 questions, 4 were answered correctly. Although the correct answers were given to such simple questions as: divide an apple between 3 children; how much will each child get? the result was measurably better than in "pure" arithemetic, in which a solution—correct or incorrect—was never even attempted.

(*b*) Arithmetical Manipulation of Objects: Monetary objects. Addition: One-half dollar, 2 quarters, 7 dimes, 3 nickels and 5 pennies were placed on the table. When asked to give the examiner 75 cents, the patient picked promptly 1 half dollar, 2 dimes and 1 nickel. In like manner he picked the correct coins for 78, 67 and 83 cents. When dollar bills were added to the former batch of money, he picked the correct bills and coins for 7 dollars and 42 cents, 8 dollars and 77 cents, etc. Generally he produced more correct than wrong performances. Also, he generally picked the correct amount of money when the amount desired was written on a slip of paper.

Various boxes tagged with a price were placed on the table. The patient was told: I want to buy this article and this one ($7 and $3). He said: "You have to pay $10." For 2 articles, priced $5 and $2.50, he asked $7.50. On the whole, he gave a considerable number of correct answers.

Multiplication: Examiner: I want two boxes like this one ($7). He first made mistakes and then corrected to $14 when he was checked. For 4 boxes of $7 each he asked $28; for two boxes of $2.50 he asked $4.60, but corrected spontaneously to $5. Mistakes and correct answers were more or less evenly balanced.

Subtraction: The examiner bought a box for $1.55 and paid with a 2 dollar bill. He returned 55 cents as change (recapitulation). For an article priced $5.25 he received $6 in bills; he returned 25 cents. Once only was the correct change of 10 cents returned when a quarter was paid for a 15 cent cigar.

Division: The patient was asked to distribute 6 quarters equally between 3 cubes. He made random attempts and soon gave it up.

Nonmonetary objects: Addition: A heap of beans was on the table. He was able to sort out 6 beans and 5 beans, and to indicate their sum as 11; 5 beans and 11 beans were added together as 13 beans; 8 and 5 beans were 14; 18 and 15 beans were 22.

Multiplication: He was asked to place 5 tablets in front of each of 3 sticks. He performed correctly and gave the product as 15. However, the product of 7 tablets added to each of 3 sticks was given as 18.

Subtraction: He counted correctly 15 beans, took off 3 and gave the difference as 13. No correct answer was given in a number of similar tasks.

Division: Tasks like the distribution of 6 beans between 3 sticks were beyond his grasp.

*Comment.*—In general, the mathematical manipulation with objects gave better results than mental figuring with objects. Monetary units were manipulated with greater facility than nonmonetary objects.

COUNTING

1. *Mental Counting.*—The patient counted from 1 to 20 correctly in five seconds. He was also able to count off any continuous series of numbers, e. g., 990, 991, 992, etc.; or 10,000, 20,000, 30,000, etc. Backward counting was always deficient.

He performed rather well in the 2-count, 3-count and 4-count. For instance, 2, 4, 6, 8, 10 was followed correctly up to 50; 3, 6, 9, 12 was followed up to 36, when he skipped members of the count and continued—partly within the count—42, 54, 62. On a second attempt he counted 3, 6, 9, 12, 14, 16, 18; i. e., he changed into the 2-count. The 4-count was given as 4, 8, 12, 16, 24, 36, 42, 54, 60.

The patient was able to correct the examiner when the latter, while counting, left out a number, even in the higher reaches of the number series.

Days and months were named correctly in the forward series, but never in the reverse series.

He indicated correctly the number of syllables in words with 2, 3 and 4 syllables when they were called out to him, in from fifteen to thirty seconds.

2. *Counting of Objects or Acts Arranged in a Continuous Series.*—A flashlight was flashed in quick succession 3, 4 and 5 times. The patient counted the flashes with only a few mistakes.

The examiner tapped the table with his fingers from 3 to 6 times in rapid succession. The patient counted correctly.

Three, 4 and 5 pills were placed in one or the other hand to be counted without looking. The patient gave many correct and some incorrect counts. (Control tests on various persons and on the examiner himself showed that normal persons make about the same type and number of mistakes in tactile counting.)

The patient counted various groups of tongue depressors correctly; he was also able to pick 4 out of 10 and promptly pointed to the second, fourth and eighth tongue depressor. He was able to separate half from a group of 4, 8 and more tongue depressors.

He was able to count correctly irregular heaps of beans or pills up to 10. But 18 were counted as 17, 21 as 24, 24 as 20, etc.

3. *Counting of Objects Arranged in Discontinuous Groups.*—He was shown 20 pills in 4 equal (homogeneous) groups (fig. 1, *4a*).

The 20 pills were then offered in similar (homeogeneous) groups (fig. 1, *4b*).

The same 20 pills were then offered in similar (heterogeneous) groups (fig. 1, *4c*).

In the same manner, all the numbers from 11 to 30 were offered in homogeneous, homeogeneous and heterogeneous arrangements, and also in straight chains. In addition, the pills were varied in color, shape and size so that of 20 pills, 6 were brown, 5 dark red, 5 yellow and 4 white. Or, 20 red pills were composed of 3 square, 6 oblong, 6 round and 5 square specimens. The result was that the patient made almost constant mistakes with any number above 10, no matter what the arrangement was with regard to size, form or color.

*Comment.*—When the 20 pills were arranged in 4 groups of fives, it would have been possible for the patient to count the first 5 and then to multiply by 4. In that case, the total count would have required less time than when the pills were arranged in groups of 5, 6, 4, 2, 3. The fact that no variation in time was observed showed that the patient always counted and never multiplied, regardless of the arrangement.

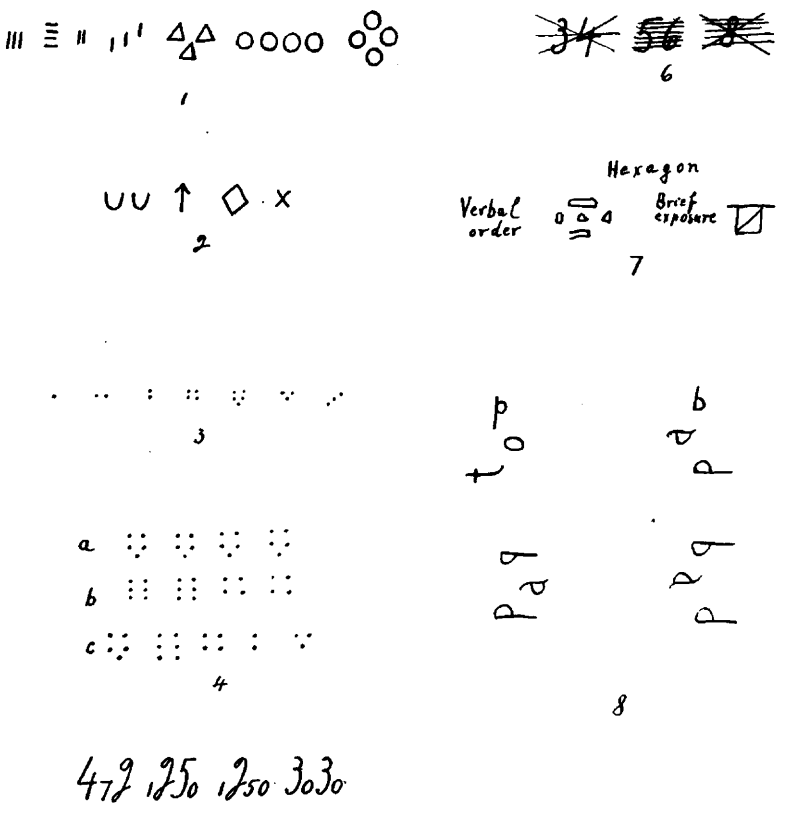


Fig. 1.—Samples of patient's ability to perform tests.

He consumed less time for small numbers and more for larger numbers: 6 pills were counted in many repetitions in an average of seven seconds; 8 pills in an average of fifteen seconds; 18 pills in an average of twenty-five seconds, and 24 pills in an average of thirty-nine seconds.

John Locke,[12] and later many philosophers, psychologists and pedagogs, assumed that the faculty for arithmetic operations is dependent on the ability to count. "A child," said Locke, "knows not that three

12. Locke, John: An Essay Concerning Human Understanding, ed. 39, London, William Tegg Company, 1875, p. 18.

and four are equal to seven till he comes to be able to count seven." Locke did not specify whether he referred to mental counting within the continuous number series or whether he included in his definition the process of counting discontinuous objects. The tests showed that the patient's capacity for mental counting was practically intact, whereas in counting discontinuous objects he was as helpless as in calculations.

SIMULTANEOUS PERCEPTION (TACHISTOSCOPIC GRASP)

1. *Tachistoscopic Reading of Numbers.*—By means of a tachistoscope with one fifteenth of a second exposure, various numbers were offered for reading. Single digits, like 4 and 7, were read promptly; 69 was read as 9-9-6; when the number was exposed a second time, it was read 6 and 9; 35 was read 3 and 5. Various other numbers with 2 digits were similarly dismembered at the first reading, but were usually read correctly at the second exposure. Rarely were they rendered correctly at the first reading.

Numbers consisting of 3 digits were always dismembered and never read correctly before the third or fourth exposure; for example, 472 was read: 1-47; 42-2; 47; 4-7-2, on sucessive exposures; 1250 was read: 2-50; 1-50; 1-0-5; 1-2-5; 1-2-5-0. When the number was finally rendered correctly after the third, fourth or fifth exposure, it was read dismembered; 1300 was read correctly as thirteen hundred; 22100 as twelve hundred, two thousand and twenty-two hundred. A number with 4 or 5 digits without a zero was never read correctly, even after 6 or 7 exposures.

No improvement of performance was obtained when the numbers were given a marked arrangement (fig. 1, *5*). When crossed out numbers (*Strichfiguren*) were given, after Goldstein and Gelb,[13] the patient was not at all embarrassed by the crosses. The crossing was done as shown in figure 1, *6*.

When the patient was directed to write down the exposed number instead of reading it, he encountered the same difficulties, dismembered the numbers and wrote the full number after several exposures only.

*Comment.*—Tests performed on normal persons with approximately the same education as the patient showed that they were uniformly able to read numbers with 3 digits at the first reading. The test therefore demonstrates that the patient lacked the faculty of simultaneous perception of 3 consecutive digits. That numbers with specially marked digits offered no facilitation and that crossed out numbers caused no aggravation of the task proved that a visual "Gestalt" factor was not responsible for the lack of tachistoscopic grasp. There was a greater tendency to grasp the digits to the left of a number than those to the right.

2. *Tachistoscopic Grasp of Signs.*—From 1 to 5 points were exposed in various arrangements (fig. 1, *3*). The patient was never able to indicate the number of points correctly, not even the number 1. Even after 6 or 7 exposures he still

13. Goldstein, K., and Gelb, A.: Psychologische Analysen hirnpathologischer Fälle auf Grund von Untersuchungen Hirnverletzter, Ztschr. f. d. ges. Neurol. u. Psychiat. **41**:1, 1918.

made mistakes. When he was asked to draw what was exposed, he invariably drew circles instead of points (of course, the wrong number of circles). Even after he was told the difference between points and circles and showed that he grasped this difference, he again substituted circles in immediately succeeding tests. When the points were exposed outside the tachistoscope, he was well able to indicate their number and to designate them as points. When dashes or small triangles or circles were offered instead of points (fig. 1, *1*), the number of the exposed figures was sometimes named and drawn correctly, even after 1 or 2 exposures, provided they were not more than 2. The dashes were even drawn in their correct spatial arrangements as horizontal, vertical and oblique dashes.

Various figures, like those shown in figure 1, *2,* were correctly drawn.

*Comment.*—The fact that the patient, after numerous trials, was unable to recognize a point tachistoscopically seems to be of particular significance. A point is, in essence, no number at all and represents, if anything, merely an element that has no spatial extension. It is, so to speak, the negation of space. Two points represent, aside from their spatial meaning, a multiple, i. e., a number in the abstract. It is also significant that a point is a whole that has no parts.

3. *Tachistoscopic Reading of Words.*—Monosyllabic words, like dog, bed and man, were offered for reading. Of 23 words, 7 were read correctly after the first exposure. Some could not be read even after repeated exposures. Others were read correctly after 2 or 3 exposures. In the series of the missed words there was always a gradual improvement in regularly progressive stages, as follows: Bed was read and simultaneously spelled out after the first exposure as d-a-d; second exposure, b-a-d; third exposure, b-e-d; man: first exposure, l-a-w-n; second, l-a-n; third, m-a-n; tip: first exposure, d-e-e-p; second, d-i-p; third, t-i-p. In other words, the last letter of the word was usually first grasped; then came the middle letter; then the first letter. It was a progression from right to left. In the words that were read incorrectly in all exposures, the last letter was almost always grasped and sometimes also the middle letter. A tendency to perseverate manifested itself in the following manner: The word tip was exposed, and the patient read d-e-e-p, d-i-p, t-i-p. Immediately afterward the word quip was exposed, and the patient read: d-i-p, d-e-e-p. Or, the word tap was exposed, and the patient read: t-o-p; immediately following, the words pad and dub were exposed; both words were read as t-o-p.

Bisyllabic words were practically never read correctly. As a rule, the last syllable was grasped or approximately grasped. For instance, fitting was read as m-o-r-n-i-n-g; bumper as h-o-p-p-e-r; letter as b-u-t-t-e-r, and bedroom as a-q-u-a-r-i-u-m. When words were offered that had some intimate relation to the patient's private life, he read them promptly and without spelling them out, even if they consisted of 3 or 4 syllables, e.g., equipment, boilermaker, foreman, Chicago, Sheehy (his name) and Anderson (his physician's name). Nonsense syllables were never grasped, even after several exposures. The last letter or the last 2 letters were usually read. There was less tendency to dismember or to spell out the nonsense words. The syllable tay, for example, was read joy, toy, d-a-y, day; nix was read mix, dix, dix; "por" was read dor, dop, p-o-o-r.

*Comment.*—The patient's inability to grasp discontinuous entities in a simultaneous act of perception was plainly demonstrated by the tachistoscopic tests. This disability was not confined to the tachisto-

scopic reading of numbers; it extended with equal force to the reading of signs and words. Moreover, there was a marked difference in the mode of operation of the spatial element with regard to reading numbers, on the one hand, and words, on the other. Numbers were read from left to right and words from right to left. The preliminary conclusion was drawn that the spatial factor, more particularly the factor of direction, played an important rôle in the perceptive activity of the patient, but that its influence was not confined to the perception of numerical entities.

### GENERAL KNOWLEDGE OF NUMBERS (ARITHMOGNOSIS)

1. *Number Reading.*—Numbers consisting of 1, 2 and 3 digits, like 9, 58, 429, were always read correctly. Numbers of higher value were read correctly when they consisted of 2 digits and zeros, e. g., 25,000 and 340,000. Reading of numbers with 3 digits and zeros usually proved unsuccessful. Toward the end of the training period the patient was able to read practically every number, even up to millions.

Abbreviations and signs were read fluently: $87.50; 40%; 9 a. m. Fractions, like 1/5, 5/90, the patient read with a terminal th, e. g., one-fifth. It must be noted that the patient was not tested for fractions until the fourth month of training.

2. *Number Writing.*—Numbers with 1 or 2 digits were correctly written to dictation. But when numbers above 100 were dictated the patient wrote them in the manner in which they are spoken: 242 was rendered as 20042; 111 as 10011; and 197 as 10097; 2,500 was written 2000500, when the dictation was two thousand five hundred, but when the dictation was twenty-five hundred, the patient wrote 2500. When the examiner wrote down 2500, the patient read twenty-five hundred; but when 2000500 was written, he read two thousand five hundred. When asked to indicate the greater number, he pointed to 2000500.

3. *Recognition of Numbers.*—The patient was shown a sheet on which were printed 54 numbers consisting of 3 digits each, like 257 and 935. The numbers were arranged in 7 vertical and 8 horizontal columns and so distributed that they could easily give rise to confusion. The number 257 of the first horiontal column was changed to 275 in the second and to 725 in the third horizontal column, etc. The patient was asked such questions as: Where is 725? Which is the largest number in the third vertical column? He answered correctly. He was then shown the following columns:

| | | |
|---|---|---|
| 126 | 652 | 1026 |
| 345 | 725 | 935 |
| 385 | 462 | 462 |
| 462 | 986 | 729 |

He was asked: In which column is 462 the largest, and in which is it the smallest figure? He answered correctly.

4. *Manipulation of Figures On Cardboard (Number Praxis).*—The patient was shown 4 pieces of cardboard, bearing the digits 5, 4, 0, 0. He read promptly five thousand four hundred. The cards were shuffled together. He rearranged them correctly on request in seventeen seconds. He performed equally well with other arrangements of 3 and 4 digits. An assortment of 25 digits was then put on the

table and he was instructed to pick three thousand two hundred and fifty. He placed the cards in the following order: 2-5-3-0-0-0, beginning from the right. When he was checked, he rearranged them as follows: 3-0-0-0-0-5-2; the 3,000 was arranged from left to right, and the 250 from right to left. One thousand two hundred and fifty was arranged: 0-5-2-0-0-1, all from right to left; when he was checked he "corrected" to 0-5-0-0-2-0-0-0-1, all from right to left. Various attempts to teach him how to avoid the mistake were without avail. *In contradistinction to what the patient did in analogous tests with letters,* to be described later, *he did not, in placing the digits, deviate from the vertical or horizontal line of conduction.*

5. *Knowledge of the Number Concept.*—The patient was asked: Which is more, 305 or 503; 1520 or 1770? He answered correctly. To guard against his tendency to recapitulate the last-named number, the questions were so modified that the greater number was named first, e. g., which is more, 875 or 524? The answers were invariably correct.

Which is bigger, ⅓ or ⅙; 1/10 or ¼? Which is more, one hour or thirty minutes, 2 dimes or 50 cents, 2 quarts or 1 gallon? The answers were correct.

What is the population of Chicago? Answer: "3 million." How much does a package of cigarets cost? Answer: "15 cents." How many cigarets are there in a package? Answer: "20."

The patient knew the difference between even and odd numbers. He was able to give his house number and to state of how many digits it consisted.

The digits 1, 8, 3 were called out to him, and he was asked to form a number out of them. He said: "12." When asked why, he answered, "1 and 3 and 8 equal 12." After explanations he said: "183." Other digits like 3, 5, 9; 9, 8, 0 and 1, 0, 7 were then correctly composed into numbers. (Note that the patient was never able on request to add correctly 3 digits as he did here spontaneously in 1 plus 8 plus 3.)

He was told: I shall call various numbers; when I come to 625 you must stop me. The examiner then called 324, 476, etc., and the patient stopped him at 625. Various other performances of this kind were correct. The examiner then wrote down various numbers and asked the patient to stop him when he wrote the number 380. He did so promptly.

He knew which number came before 19, 34, 216, etc.

He was given lengthy instruction (after Peritz [8]) that the digit 1 contains one vertical bar; 3, two semicircles; 4, one cross bar and two vertical bars; 5, one cross bar, one vertical bar and one semicircle, etc. He then promptly indicated that the 6 has one semicircle; 7, one cross bar and one vertical bar, etc.

*Comment.*—The patient's knowledge of the number concept must be called fair, if not good. The mistakes that he made were again prominently correlated with a factor pertaining to a spatial element. He could not grasp that the 1 in 197 has an entirely different value from the 1 in 21. That this "position value" of digits constitutes a spatial element needs no explanation. The inability to manipulate spatial arrangements was particularly evident in his manner of arranging various numbers from right to left and vice versa.

ESTIMATION OF MAGNITUDES AND QUANTITIES

1. *Objective Estimates.*—Of 6, 8 and 12 tongue depressors, arranged in 3 piles, the patient designated correctly and promptly the largest, second largest and third largest groups. When 7 tongue depressors were placed in the patient's left hand and 9 in his right hand, he said correctly that he had more in his right hand. He filled, on order, a 2 ounce glass with water up to the mark "2 ounces." He compared various glasses of equal size filled with different amounts of water and named correctly the glass with more, less water, etc., ordering them correctly as to their quantity.

The examiner placed a pile of about 200 pills on the table and separated a heap of 60 pills. The patient was asked to separate the same number of pills from the remaining heap; he matched them by 55 pills, merely by sorting without counting. A heap of 26 pills he sorted, without counting, into groups of 13 and 13; 30 pills he divided into 12 and 18; 42 pills into 19 and 23, and 46 pills into 23 and 23. The order was: Divide this heap into 2 equal parts. When larger numbers were offered, he made gross mistakes, as compared with normal controls. For example, he divided 235 pills into 152 and 83 and 152 pills into 64 and 88.

While in all the preceding tests the factor of magnitude predominated over that of quantity, the latter factor predominated in the following tests: twenty-four pills were placed in a small box and 24 pills in another box of the same size. The patient designated them as "pretty near tied." They were put together in one box, and he was asked, how many are they? He said: "about 50." Thirteen pills were so placed in a small box that they overlay each other. He indicated their number as "about 14."

2. *Mental Estimates.*—The patient estimated the length of his middle finger at 3 inches, that of his leg as "about 60 inches," the duration of a dinner as ten minutes, that of a trip downtown as of "at least an hour" (correct), and that of a trip to New York as twenty hours. The speed of an automobile he placed at 50 miles, that of a train at 60 miles and that of an aeroplane at 120 miles.

*Comment.*—On a priori grounds, it is to be assumed that a successful manipulation of arithmetical problems presupposes a facility for estimating quantities and magnitudes. For example, if one wishes to compute how much the sale of 100 cows will net, one must not state the result in terms of pennies or of a few dollars. One must immediately focus on numbers involving thousands of dollars. The tests showed that the patient was fairly well able to place himself, by estimation, in the approximately correct neighborhood of the expected magnitudes and quantities. A gross inability to effect correct estimates could therefore be ruled out as a possible cause for the difficulties in arithmetic.

MEMORY

The patient was well able to repeat 3 numbers, 3 proper names or 3 simple words in immediate reproduction. After three minutes he usually failed with the numbers, even if they consisted of 1 digit each, but reproduced 1 or 2 of the proper names and words. After six and nine minutes he usually failed in numbers, words and names. Nonsense syllables he could not repeat at all, except when they were offered 1 at a time. The sentence "I visited George on Monday and Joe on Friday" was repeated: "John visited George on Monday and Joe on Friday."

He repeated correctly the following sentences: "A street car goes east, an elevated north and an automobile west." "I eat at the Palmer House on Monday and at the Congress Hotel on Wednesday."

When asked to repeat sentences that contained numbered objects, the patient repeated the objects correctly but not the numbers. "A man has 250 horses and 30 cows" was reproduced as "A man has 50 horses and 25 cows." The sentence "his brother has 85 horses and 250 cows" was rendered: "he has 250 horses and 200 cows" (mark the recapitulation of 250). To ascertain whether he conceived the general meaning of the implied relations, the same sentence was offered in the following version: "A man has 25 horses and 30 cows. Is he wealthy? The answer was: "He is just making a living." He was then told: "His brother has 85 horses and 250 cows. Is he wealthy?" he said: "Yes."

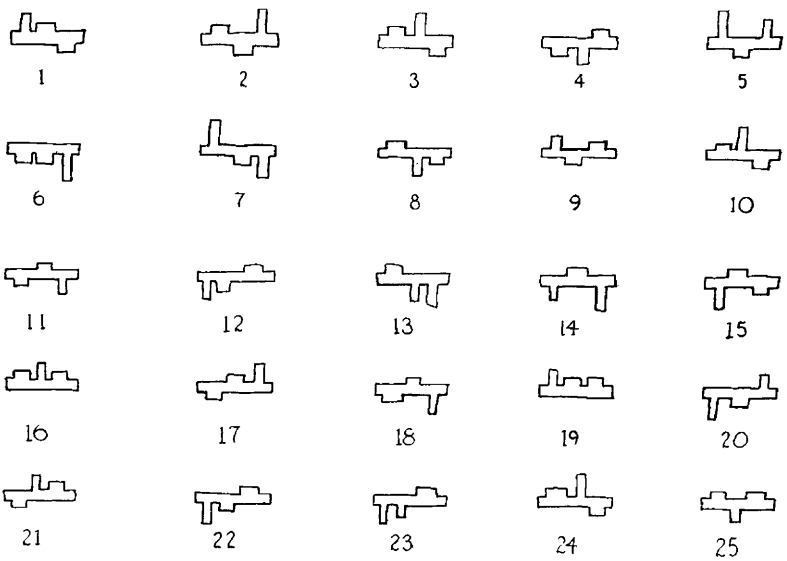


Fig. 2.—Twenty-five designs shown patient as a test for optic memory.

The patient was given a sheet with 42 numbers and told: Keep 12 and 14 in mind and underscore them on this sheet. In four 12's and four 14's he made 1 mistake.

He was given (after Poppelreuter [14]) a sheet of paper with 25 simple designs drawn on it (fig. 2). The examiner had the identical 25 patterns cut separately in his hand and exposed them severally for about three seconds. The patient identified the corresponding designs on the sheet in an average of twenty-two seconds.

*Comment.*—That memory enters as a prominent factor into the execution of mathematical problems needs no discussion, as it forms an

14. Poppelreuter, W.: Psychologische Begutachtung der Erwerbsbeschränkten, in Abderhalden, E.: Handbuch der biologischen Arbeitsmethoden, Berlin, Urban & Schwarzenberg, 1928, vol. 6, pt. 101.

integral part of every mental activity. The tests showed that the patient was able to repeat isolated numbers. But numbers in their mathematical context (25 horses and 30 cows) he could not repeat. In other words, his immediate retention was preserved for nonmathematical elements and for isolated numbers, but was practically abolished for numbers within their mathematical context. The conclusion was drawn that one of the factors that accounted for the patient's difficulties in manipulating mathematical problems was his defective immediate retention for numbers in their mathematical context.

### CHRONOGNOSIS (CONCEPTION OF TIME)

1. *Reading Time from a Watch.*—The patient was able to read correctly the half hours only, 2:30, 11:30, etc. He always said half past eleven or half past two and never eleven thirty or two thirty. No other time relation was read correctly. There was a tendency to confuse symmetrically opposed time markings on the dial; for example, he read a quarter past 12 instead of 11:45 or 12:00 instead of 6:00 or a quarter to 9 instead of 3:15. When he was asked to indicate on a dummy watch the same time that the examiner showed him on his own watch, he made very few mistakes with the half hours 3:30, 10:30, etc. In all other time relations he made the same types of mistakes as in reading from one watch. The same results were obtained when the time was written on a slip of paper and the patient was instructed to arrange the hands of the dummy watch accordingly.

2. *Estimation of Time.*—The patient was asked, which is more, three minutes or twelve seconds; two months or five weeks? He gave correct answers. Contrary to expectation he did not recapitulate the last number. He estimated correctly the examiner's age. He was able to tell approximately how long he had been at the hospital and in what month and on what day he had arrived there, and he was well able to name the current day and month.

3. *Reproduction of Time Data Belonging to General Information.*—The patient knew that Tuesday came before Wednesday and March before April. Christmas he placed on the twenty-fifth day in January; Easter "somewhere in April." He knew his age and the date of his birth. He was able to give in detail the date on the day of this examination as "Monday, January, 1930, about the 8th."

*Comment.*—The patient had a fair ability to estimate time and to reproduce time data referring to general knowledge. His deficiency in reading the time from a watch was in rather marked contrast with his general understanding for time concepts. Here again, however, the elements of spatial orientation and of immediate retention of numbers within a context seemed to play an important rôle. The spatial element came prominently into play when he confused symmetrically opposed time markings, like 12:00 and 6:00. Since we knew from previous tests that simple numbers, like 6 and 12, did not appreciably tax his retentive memory, the inability to distinguish these numbers on a watch must be attributed solely to the spatial and contextual elements.

LETTER, WORD AND SENTENCE GNOSIS

1. *Manipulation of Letters and Words.*—The small letters a to g were placed on the table and the patient was told to arrange them in their proper order. He was always able to do this correctly. *However, most of the letters were placed either above or below the horizontal line of conduction; furthermore, there was a tendency to place the letters at various angles of deviation from the vertical axis* so that the letters were set obliquely or stood upside down or were parallel with the horizontal instead of the vertical line (fig. 3). The same types of misplacement were observed with the capital letters (fig. 3) of the alphabet and with the numerals 1 to 9 (fig. 4). Analogous mistakes in spatial arrangement were made when single letters were to be combined into words or words into sentences. The

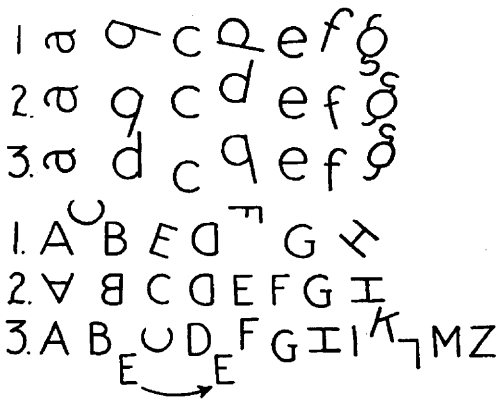

Fig. 3.—Arrangement of small and capital letters made by patient.

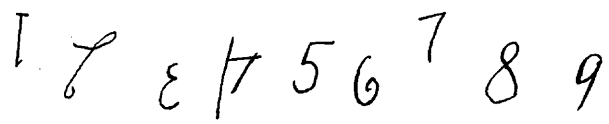

Fig. 4.—Arrangement of numerals 1 to 9 made by patient.

order of succession was thereby always found correctly, but the spatial arrangement with regard to the horizontal and vertical was always distorted. For example, when the patient was given the letters p, o, t, with the request to arrange them into "pot," or b, a, d, he arranged them as shown in figure 1, *8*. The succession was again preserved.

Reversals of the initial and terminal letters were frequent, but the central letter was always left in the center. The patient was asked to arrange the letters p, o, t into "pot" and reversed them into t-o-p. When the letters p-o-t were placed before him, he sometimes read them as "pot" and sometimes as "top." Similar mistakes, both in arranging and in reading, were made with b, a, d and g, o, d. With nonsense words, like "mun," "nom," and "mok," he made the same types of mistakes.

Three each of the letters b, d, p and q were placed on the table, and the patient was directed to pick out the letters to order. He made few mistakes in numerous tests.

2. *Writing.*—Whether from copy, from exposure or to dictation, writing was reduced to an illegible scribble in the first few weeks of the testing period. This scribble was always placed on the right half of the sheet of paper (fig. 5). The patient would start in the middle of the sheet and write one word; the next word would be placed on the next line a move farther to the right. In the next line the word was placed still farther to the right, etc. He was not able to maintain the line of conduction. The displacement to the right and the missing of the line of conduction persisted for many weeks, even at a time when he had improved both in caligraphy and orthography. After he had improved under daily training, his writing exhibited the following types of defects:

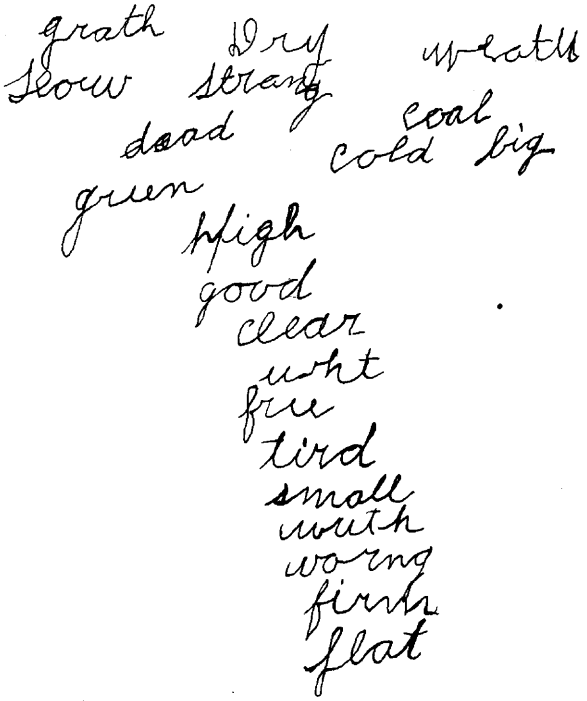


Fig. 5.—Sample of arrangement of patient's writing.

(*a*) Writing from copy: The examiner wrote the sentence "Mary picked one poppy" on a slip of paper. The patient copied "Mary kicked kokey." In other words, the k in the middle of the word "picked" was anticipated and placed at the beginning (ante-position); afterward, the k was recapitulated and "kokey" written instead of "poppy." Other instances of antepositions were: "thoes" for those; "Staurday" for Saturday; "Saturady" for Saturday; "umberla" for umbrella; "Sepmebrer" for September, and "nan" for man. When "howo long" was written for "how long," the mistake might have been an anticipation of the o from the succeding "long" or a recapitulation from the preceding "how." An undoubted recapitulation took place when "I have a hook" was written for "I have a book."

In some of the copied sentences the words were juxtaposed without an intervening interval of space, e. g., "wewriteinthisway"; "thismanisreading"; "chicago-

ill"; and "whatisthis." In other sentences an elision of letters occurred, so that the words were not merely juxtaposed but actually fused together. Such fusions were "wherarey" for "where are you"; "wherdoulive" for "where do you live," and "howlng is thfirst line" for "how long is the first line." Elisions of letters in single words were frequent; the word "studying" was copied as "studing," "stdying" and "stying"; "September" was copied as "Setember," "Septmber," "Sepember"; the word "Saturday" was copied "Saurday," "Surday," "Sturday."

(*b*) Dictation: The anticipations, juxtapositions, elisions and recapitulations were not as frequent as in writing from copy. Most of the mistakes were orthographic in nature. Monosyllabic words yielded the relatively best performance, regardless of whether they were nouns, adjectives, verbs or prepositions. In bisyllabic words the mistakes became more numerous, and polysyllabic words were practically never written correctly. In order to discover a possible regularity in the type of mistakes, the same word was offered on different days. However, the same word furnished different mistakes on different days. Thus, profit was once written as "proffith" another time as "boffth"; teacher once as "thechet," then as "checer"; business as "biaas" and "bisos," and practical as "practle" and "prirelh."

(*c*) Writing from Exposure: Words and sentences were exposed for about thirty seconds. The mistakes were practically the same as in writing to dictation.

3. *Reading.*—The patient read fluently any kind of material, sense or nonsense, letters, syllables, words, sentences and paragraphs. He was able to read a word, like CHICAGO both in a reversed and in a vertical arrangement. He also read sentences in which words were juxtaposed, like: IWENTTOTHESTORETO-BUYSOMETHING. The sentence "The big dog ran out of the house" (Dearborn[15]) was given the patient in mirror script. He was unable to read it.

*Comment.*—That the main mistakes noted in writing had in common an inability to handle certain spatial elements seems obvious. The anticipations and juxtapositions certainly belong to this spatial category. That the displacement to the right, the deviation from the line of conduction and the reversals of the initial and terminal letters are due to some sort of lack of spatial recognition goes without saying.

### DRAWING

The patient was asked to draw objects like an apple, a pear, a shoe, a church, a pipe and a cigar to verbal order. The examiner then drew each object on a piece of paper, exposed it for about thirty seconds and the patient reproduced it. Then, the object was placed both as such and as a drawing on the table, and the patient was asked to copy it. The results, as shown in figures 6, 7 and 8, may be summarized as follows: There was slight improvement in the patient's copying as compared with his drawing after brief exposure or to verbal order. The performance was generally better with objects and patterns that had a symmetrical arrangement (house, bottle, table, box, apple, human face) than with objects and patterns that were more or less asymmetrically arranged (church, shoe). Complex drawings like a hexagon he could not reproduce in spite of their marked symmetry (fig. 1, 7).

15. Dearborn, W. F.: The Etiology of Congenital Word Blindness, Harvard Monographs, Educational Series 1, June, 1925, vol. 2, no. 1.

It was always ascertained that the patient knew the use and meaning of the objects and patterns he was asked to draw. In his former occupation he had had to make sketches of the jobs he superintended. With such figures as a hexagon he was familiar.

He was able to draw a line between two points and to connect three points into a triangle. When a line was drawn on paper by the examiner, the patient was well able to halve or double it.

In drawing a ground plan of the examining room, on request, he missed the proportions of the furniture and their location relative to each other and to the room, and inserted 3 doors instead of 2.

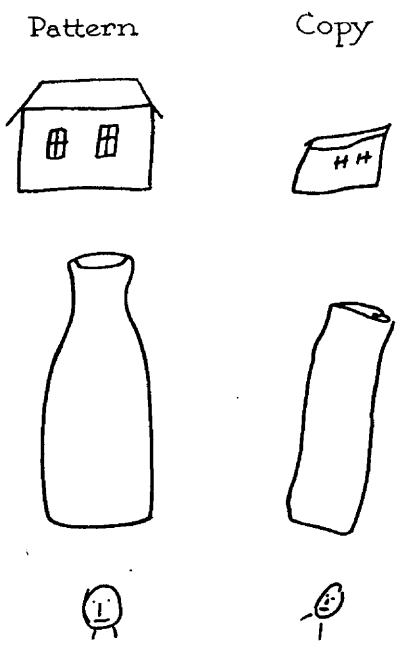


Fig. 6.—Drawing after pattern.

When the object or pattern to be copied lay perpendicular to the patient's body, he invariably drew it inclined to the right. He was unable to draw any straight line, whether horizontally or vertically, and always deviated to the right of the vertical and above or below the horizontal line. This deviation took place regardless of whether he started out from above or below or of whether he drew the lines by themselves or in the context of another figure.

In spite of constant attempts to correct his drawing, practically no result was obtained throughout the period of training.

*Comment.*—That the spatial element is all important in drawing is understood. The tests seem to prove that this spatial element is prominently associated or perhaps identical with the factor of symmetry. What requires particular comment was that drawing from copy gave no appreciably better results than drawing from memory. In other words, the defect was such that visual facilitation gave little aid. It is

also worth noting that the results were no better when the patient copied from the actual object instead of from the examiner's pattern. The deviation from the vertical and horizontal line is a feature that has already been noted in the tests for writing.

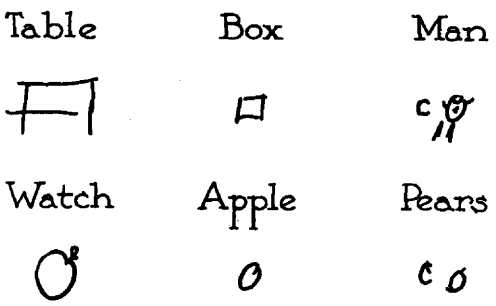


Fig. 7.—Drawing on verbal order.

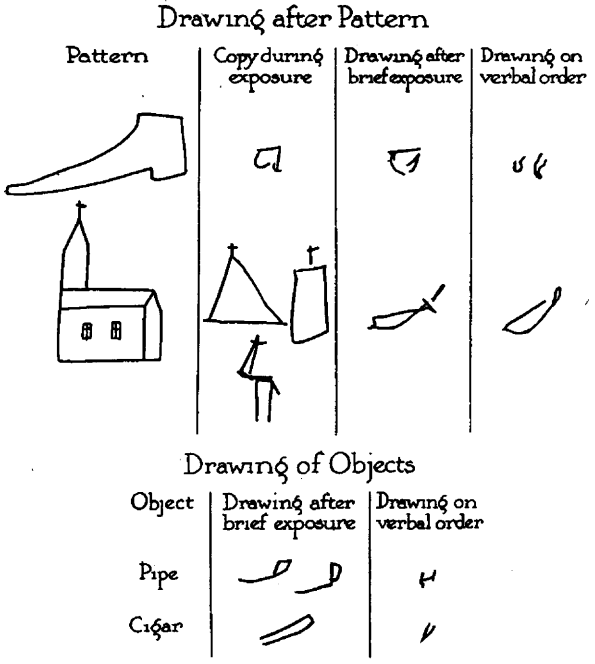


Fig. 8.—Drawing after pattern and on verbal order.

## SPATIAL GNOSIS

1. *Manipulation of Objects in Their Spatial Relations.*—Small sticks were placed on the table in various arrangements, as suggested by Strauss [16] in his study of "constructive apraxia." In most cases the patient was unable to reconstruct the

16. Strauss, H.: Konstruktive Apraxie, Monatschr. f. Psychiat. u. Neurol. **56**: 65, 1924.

pattern. While he manipulated the sticks the original pattern was left exposed on the table. The relationship between pattern and copy is plain in figure 9.

Seven tongue depressors were arranged in parallel groups of II III II and then thrown together. The patient rearranged them as III II II. In many repetitions with 7, 8 and 9 tongue depressors he made constant mistakes.

Triangles, squares and quadrangles of various sizes were cut out of paper. When asked to cut similar figures, the patient was at first helpless, but he performed well after due explanation. After he understood the task he cut the figures correctly on plain order without being shown a pattern.

Both on plain order and after demonstration, the patient was able to cut away the upper corners of a square, the left upper corner, or all the corners together. On order and after pattern he cut a triangular notch in the lower, upper or lateral sides of a square.

Six triangular, rectangular, quadratic and pentagonal "windows" were cut out of a piece of cardboard. The patient was asked to put the "windows" back into their frames. After initial mistakes he performed well in many repetitions. The

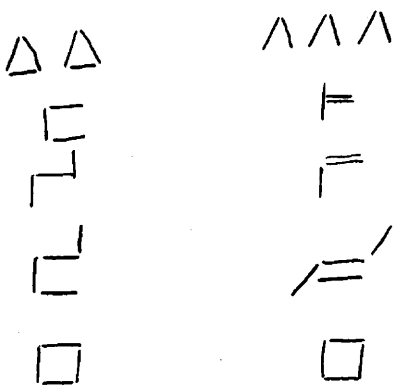

Fig. 9.—The left column of designs represents the patterns; the right, the performance of the patient.

"windows" that fitted the frames were then mingled with inlays that were either too large or too small or were differently shaped (hexagons, octagons, etc.). When asked to pick out the fitting "windows" he performed correctly.

2. *Spatial Orientation in Familiar Surroundings.*—(*a*) Spontaneous Orientation in Space: The patient did not require supervision in his movements at the hospital. He was always able to find and to replace his belongings in their proper places, had no difficulty in moving from room to room or from floor to floor, and superintended his own private affairs in town without assistance. He made use of street cars and elevated trains without difficulty.

(*b*) Directed Orientation in Space: The patient was shown a map of Chicago and was able, on order, to point to the loop, to the Municipal Pier and to the approximate location of the street and house where he lived, and he made no mistake in indicating east, west, south and north.

*Comment.*—The patient performed fairly well in all the tests for spatial gnosis, except when he was asked to arrange sticks into a figure or to arrange a number of tongue depressors in prescribed groups.

APPERCEPTION

1. *Reproduction of Analytic and Synthetic Data Belonging to the Common Stock of General Knowledge.*—The patient was fairly well able to spell simple words, to indicate the number of syllables even in 4 and 5 syllable words and to name a string of words that begin with A or G, etc.

He gave an adequate account of the difference between various notions (fear and respect) and objects (pen and pencil). The difference given was usually pertinent.

The patient named opposites promptly and correctly for nouns, adjectives, verbs, prepositions, etc. (man, woman; rough, gentle; to run, to sit down).

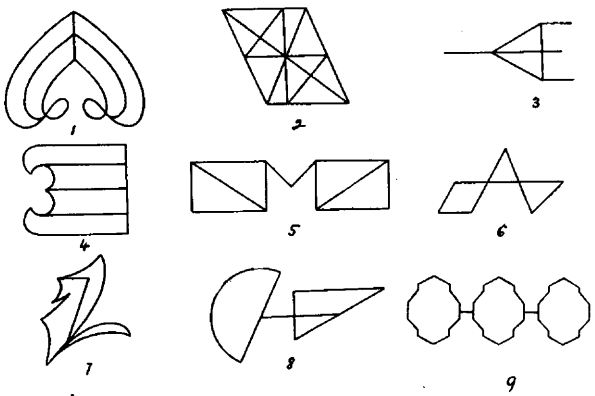


Fig. 10.—Figures shown patient to find letters and numbers.

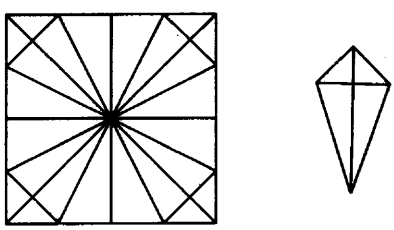

Fig. 11.—The whole object and the part figure.

He enumerated with facility the parts of a whole (genus and species) and the varieties of trees, fruits, birds, colors and religious creeds that he knew.

The patient promptly offered synonyms for policeman (guardian of the law), for pants (trousers) and for automobile (motor car).

2. *Thought Processes Involving Logic and Common Sense.*—(*a*) Logical Inference: He was asked: A fly is an insect. What is a woodpecker? Answer: "A bird." What is a horse? "An animal." What kind of an animal? "One can use for working." He was told: Every human being has 2 legs. I am a human being. Hence, I have 2 legs. He was then told: Every watch shows the time. This is a watch. What follows? He answered: "It tells the time." In various other syllogisms he had no difficulty in finding the conclusion from the premises. He recognized absurdities promptly and corrected them.

(*b*) Psychologic Processes: The patient gave a good detailed description of what he intended to do in order to get the telephone number of a man who owned a restaurant but was not yet listed in the directory. He outlined well in detail how he would go about getting a house built after he had saved $3,000.

3. *Comparison and Abstraction.*—The patient was shown a card with 8 straight and 9 curved lines and was well able to designate them correctly as curved or straight. He was also well able to point out the smaller, larger and largest angles on a card on which were drawn acute, right and obtuse angles of various sizes.

The patient was able to indicate in *9* of figure 10 the 2 Hs; in *8*, the reversed D; in *5*, the M, and in *3*, the E. The 4 in *8* he failed to detect. But the 6, 3 and 7 in *1*, *4* and *7* he found promptly. In all these tasks the question was: Show me an H, E, M, etc., in this figure. He was shown *2* of figure 10 and asked: "Can you find letters in this figure?" He isolated N and V. He was asked to show the part figure in the total figure 11, and did so promptly. All the tests were modifications of those found in Buehler's [17] book.

4. Interpretation of Pictures: The patient described primitive and complex pictures, sometimes synthetically, but as a rule analytically. He often held the picture upside down while describing it, but furnished a fairly competent description of the details.

*Comment.*—The apperceptive processes were so well within the patient's grasp that they could be safely ruled out as factors responsible for the defect in arithmetical operations.

## GENERAL COMMENT

Most authors who have studied acalculia have attempted to reduce the defect to a fundamental inability to handle some spatial element. Lange,[8] after a careful study, even ventured to specify the precise type of spatial element. He said that the errors made by his patient can all be explained on the basis of an inability to manipulate the "category of direction in space." He added that the concepts of quantity and magnitude are "numbers associated with spatial conceptions." The term "category of direction in space" he seemed to identify with the "conceptual and creative structuralization of space" (begriffliche and schoepferische Gliederung in Raume). These terms are quoted verbatim in order to show the vague and indefinite conceptions that have been evoked to interpret the nature of acalculia. On the basis of our material we agree with Lange and others that some sort of spatial element is a conspicuous factor in determining the defect; we also accept the dictum that the concept of direction plays a prominent part in this spatial element; but that the inability to handle this spatial element is solely responsible for acalculia is an assumption unwarranted by our observations. Although in the present study we may have disclosed some new facts, we do not feel justified in claiming that we have established

17. Buehler, K.: Die Gestaltwahrnehmung, Stuttgart, W. Spemann, 1913.

the primacy of any one of the several factors that entered into the causation of the acalculia. The problem at present needs, not the premature elaboration of an intriguing theory, but a refinement of the methods of examination, a further accumulation of case reports or both.

In attempting to analyze our results we shall limit the discussion to functions in which the patient made no or little progress in spite of intensive training. These functions were: addition of numbers the sum of which exceeded 10; subtractions and divisions; tachistoscopic reading; writing; the ability to grasp the meaning of the "position value" of digits; counting of discontinuous quantities; construction of simple figures from small sticks or tongue depressors; constructing numbers from single digits on cards; retention of numbers above 10 within a context; reading the time from a watch; drawing of any kind.

*Construction of a Continuous Total from Discontinuous Parts.*—It is easy to postulate that all these functions require the facility to construct a continuous total out of discontinuous parts, and to say that the inability so to construct was the sole factor that underlay the defect. This interpretation, however, is not sufficient. The patient was unable to read monosyllabic words tachistoscopically; yet he mastered without difficulty all kinds of words, sentences and paragraphs in plain reading. In plain reading he was required to "construct a continuous (word or sentence) total out of discontinuous (letter) parts," and if the construction disability was at the root of the defect, why did not this inability make itself evident in plain reading? Again, he was unable to construct the total of 77 out of the parts 50 plus 25 plus 2. Why then was he able to construct the total of 77 cents out of the parts one-half dollar, 1 quarter and 2 cents?

*Spatial Direction.*—Our patient was incapable of maintaining the line of conduction when he constructed the word "bad" out of cards bearing the separate letters b, a, d. Yet, in constructing the number 3,250 out of corresponding single digits, he made mistakes with the "position value" but did not deviate from the line of conduction. In tachistoscopic reading of words he grasped the right letters or the right syllables of a word first; i. e., he reversed the customary direction of reading from left to right. Why then did he read numbers, likewise offered tachistoscopically, from left to right, an observation which we believe has not previously been made? Last [18] and Heidenhain,[19] who used the tachistoscope, did not deal with acalculia. The difference

18. Last, S. L.: Ueber eine Störung der optischen Formauffassung, Monatschr. f. Psychiat. u. Neurol. **76**:238, 1930.

19. Heidenhain, A.: Beitrag zur Kenntnis der Seelenblindheit, Monatschr. f. Psychiat. u. Neurol. **66**:61, 1927.

cannot be explained on the basis of a "body schema" (Schilder [20]), according to which it might be said that the left side of the body did not exist for him (a popular way of expressing this would be by saying "he favored the right side"). It is true that he could not draw a line from right to left, but always drew from left to right. If this was due to a "loss of the left side," why was he not able to pull the bed sheet to the right? Morever, why did he read numbers tachistoscopically from the left? These objections show that the factors of construction and direction, while undoubtedly playing an important rôle, do not explain entirely the essential mechanism of the defect. Some other factor must either be added to or replace the two factors mentioned.

*Spontaneity and Automatization.*—A striking observation was that a function that was ordinarily beyond the patient's span was nevertheless sometimes correctly exercised when he was unaware of exercising it. He was unable, for example, to solve the problem $9 - 6 = 3$, yet this subtraction was carried out when it was substituted as an answer to the problem $9 + 6 = 3$. Again, he was never able on request to add together 3 digits correctly; yet when the single digits 1, 8 and 3 were called out to him and he was asked to form a number from them he promptly added them together as 12. In other words, identically the same problem was solved under conditions of spontaneity and was almost regularly missed under the conditions of a "task." What then, it may be asked, is the difference between a spontaneous performance and one imposed as a "task"? Wundt [21] called the one "actively intended" and the other "passively experienced" (aktiv gewollt und passiv erlebt). This is a translation rather than an explanation. McLellan and Dewey [22] seem to have had a similar active principle in mind when they stated: "Number represents a certain interest, a certain psychic demand; it is not a bare property of facts." Whether it is called "active intention" or "psychic demand," it is obvious that a certain principle, somewhat in the nature of a special strain, becomes operative in a problem when it is offered as a "task" and becomes proportionately reduced when the same problem is dealt with spontaneously.

This difference between the spontaneous performance and a "task" explains many but not all of the mistakes made by the patient. Under training he mastered the multiplication table but remained inefficient

20. Schilder, P.: Vestibulo-Optik und Körperschema in der Alkoholhalluzinose, Ztschr. f. d. ges. Neurol. u. Psychiat. **128**:784, 1930.

21. Wundt, Wilhelm: Grundzüge der physiologischen Psychologie, Leipzig, Wilhelm Engelmann, 1903, pt. 5, p. 332.

22. McLellan, J. A., and Dewey, J.: The Psychology of Number, New York, D. Appleton and Company, 1895, p. 21.

in analogous additions. In terms of "constructing a total out of its parts" it should be largely immaterial whether the total 42 is to be constructed out of the parts 6 times 7 or the total 13 out of 6 plus 7. As a matter of construction the problems are practically alike. Viewed under the aspect of spontaneity, however, the two problems differ essentially. Multiplications of numbers are much more easily automatized than additions of the corresponding numbers. Whether this greater liability to automatization is an intrinsic quality of the multiplication process or whether it is due to the fact that traditional school training favors repetitive exercises in the multiplication table may be left unargued. The fact is that perhaps under the system of school training the multiplication table has a greater tendency to become automatized than have additive problems.

The fact that our patient was hardly able to count a number of beans in excess of 10 but exhibited a normal facility for mental counting even up to millions points in the same direction. Mental counting is certainly a labored "task" for the early school child, but becomes automatized through uninterrupted practice, while counting of objects is hardly ever taught and hence is not automatized. Again, the addition of one-half dollar, 1 quarter and 2 cents becomes automatized in daily life, whereas this is not true for the addition of 50 plus 25 plus 2.

Additional evidence of the fundamental difference between an automatized performance and an unfamiliar "task" was furnished by the mistakes made in tachistoscopic reading of words. The patient made many mistakes with simple monosyllablic words; but such polysyllabic words as boilermaker, equipment, foreman, Chicago and his own and the physician's name he read promptly. All these words were familiar to him, or automatized; they did not involve a strained "task." Another observation that accentuated the facilitation gained through automatization was: The patient was never able to read monosyllabic nonsense words from the tachistoscope, but was sometimes able to read monosyllabic sense words. While in all these performances the factors of direction and construction play an unmistakable part, the factor of automatization must also be considered. But even these three factors combined are insufficient to explain all the mistakes made by the patient. For example, the difference in the direction of reading numbers and words, respectively, from the tachistoscope cannot be accounted for on that basis.

*Analysis and Synthesis.*—Another factor that must be given due consideration in the evaluation of the acalculia is the patient's relative proficiency in handling an analytic problem and his relative deficiency in manipulating a synthetic problem. When he was offered words

and numbers to read from the tachistoscope, the task was to grasp a word or a number total, i. e., to synthesize letters into a word and digits into a number; instead, he dismembered the total and failed to synthesize the component parts. For instance, he read "top" as t-o-p or 69 as 6-9. This tendency to grasp parts only was perhaps responsible for his inability to recognize points and to indicate their number when they were offered tachistoscopically. He always designated and drew points as circles and invariably named the wrong number, even when 1 point only was offered. The corresponding numbers of triangles, circles or dashes were read with considerably less difficulty. It may be suggested that the inability to read points and to indicate their number perhaps was due to the fact that a point is the only geometric entity that cannot be decomposed into parts. The patient's general facility for analyzing parts out of wholes, in contradistinction to his inability to synthesize, was convincingly demonstrated by the promptness with which he read the letters M, E, A, etc., and the numbers 4, 7, 6, etc., out of the integrated totals of the figures in figure 10. A similar case of manipulating analytic tasks was apparent when the patient described a 5 as consisting of a vertical bar, a horizontal bar and a semicircle and a 7 as consisting of a horizontal bar and a vertical bar.

*Symmetry in Space.*—Considerable light was thrown on the principal mechanisms that entered into the patient's defect by his performances in drawing. That he drew relatively best in copying, worse after brief exposure of a pattern and worse yet in drawing to verbal order can be explained by the proportionately decreasing degree of visual facilitation of these tasks. The almost constant deviation to the right emphasized his difficulty in handling the factor of direction; the defective rendering of form, contour and relative size pointed to an inability to construct totals out of parts, each separate stroke of the pencil representing a part of the total drawing. But a striking observation was that his drawing improved in direct proportion to the degree of symmetry of the pattern. Thus, he was helpless in drawing a shoe, failed in designing a ground plan of the examining room and copied the pattern of a church with little regard to the size, form and mutual arrangement of the parts. On the other hand, his performance was fair in drawing a bottle and a house and was almost satisfactory in the sense of construction when he copied or drew on verbal order a box, a table, an apple, a pear, a watch or a human face. In other words, he failed with objects (shoe, ground plan and church) in which the symmetrical element was complex and difficult to trace and performed fairly well with objects which offered good symmetrical orientation (round and quadratic objects).

The factor of symmetry must be given particular weight in the discussion of acalculia. All the elements that have been previously mentioned—direction, construction, analysis and synthesis—are ultimately related to the factor of symmetry. The essential feature of symmetry is either a point or a plane of reference about which the parts of the total are more or less symmetrically or asymmetrically arranged. Construction and direction are both oriented on such a point or plane of reference. Synthesis and analysis are identical with composition and decomposition, and are ultimately guided by relation to a point or plane of reference.

*Natorp's Theory of Calculation.*—The introduction of the concept of symmetrical or asymmetrical reference offers a possibility to link up the patient's acalculia with the general spatial agnosias that he displayed in all functions that require a grasp of direction, construction, synthesis and analysis. In order to effect this linkage we shall refer to a theory of calculation that was formulated by Natorp [23] in 1910. Since this theory is complex and elaborate, we shall content ourselves with merely outlining it and refer the reader to the original for fuller information. Natorp proceeds from the hypothesis that counting and calculation alike have as their point of departure (point of reference) the zero. Even if one begins to count from 20, 21, 22, etc., he has always in mind the fundamental starting point of zero, and knows that the number 20 is 20 points distant from zero. The universal function of zero is that of a general point of reference. Additions, Natorp continues, are a particular type of counting, a so-called discontinuous count, or a combination of variously counted entities. In order to calculate that 3 plus 2 equals 5, one proceeds from zero and counts 1, 2, 3; then one interrupts the counting process and returns to the zero; now one starts again, counting 1, 2, all the while keeping the first count of 1, 2, 3, in mind. By keeping both counts in mind, one knows that, proceeding from the zero, he has counted first 3, then 2 points or altogether 5. The uniform progression in a forward direction from the zero contains the element of direction; the combination of the separate counts of 3 and 2 and their unification in the act of keeping the counts "in mind" contain the element of the "construction task." The latter requires both a process of keeping the various counts separately (analysis) and the process of keeping them together in mind (synthesis). Thus, the three processes that have been discussed as constituting at least a part of the defect in our patient are actually seen to enter the process of simple addition.

23. Natorp, Paul: Die logischen Grundlagen der exakten Wissenschaften, Leipzig, B. G. Teubner, 1910, p. 132.

Natorp explained subtractions, multiplications and divisions on the same basis of various processes of counting, proceeding severally from the zero as a point of reference. We do not feel competent to judge the merits of Natorp's theory of calculation, and state merely that we are inclined to agree with his thesis. If the theory is correct it offers a fairly satisfactory basis for explaining on a common ground most but certainly not all of the patient's defective performances, both in calculation and in the other functions he was incapable of exercising. One common ground on which these functions meet is the inability to proceed from a given point of reference and to remain focused on it, progressing with the task of construction.

## SUMMARY AND CONCLUSIONS

1. As a result of carbon monoxide poisoning, a patient developed a condition the main residual symptoms of which were acalculia and agraphia.

2. The main factors that seemed to enter into the causation of the acalculia were: difficulty of constructing a continuous total out of discontinuous parts, difficulty of following a given direction from a given point of departure and difficulty of grasping a whole without decomposing it into its parts.

3. These spatial and construction elements alone, however, do not account for the defects without taking into account the additional elements of the "task" and of symmetry. Even then it is impossible to explain all the defects observed.

4. Natorp's theory of calculation offers some but not an entirely satisfactory explanation of the nature of the defects.

5. The need for a refinement of testing methods and for an accumulation of case reports is stressed.

## ABSTRACT OF DISCUSSION

DR. PAUL SCHILDER, New York: This paper has demonstrated one type of acalculia, which I should like to call the optic type. One is often able to demonstrate agnosic difficulties with the tachistoscope when one cannot demonstrate them with other methods. But I wish to draw attention to the fact that it is only one of the possible types of acalculia. There is another type of acalculia which is combined with finger agnosia and with agraphia, and that is a different as well as an important type, especially since it is known that its localization is more toward the parietal region, whereas the type demonstrated is a purely optic type.

In every case of acalculia one should be especially interested in the way the patient writes, because in that way one will determine whether there is finger agnosia or agraphia also.

DR. E. D. FRIEDMAN, New York: Recently, we had occasion to observe a patient at Bellevue Hospital who attempted suicide with illuminating gas and who, on emerging from his stupor, exhibited an agnosia in the auditory sphere. He presented a type of word deafness similar to that encountered in advanced lesions of the temporal lobe. In this case it was assumed that there were bilateral lesions in the cortex with resulting auditory agnosia.

DR. DONALD GREGG, Wellesley, Mass.: Was Dr. Singer's case complicated by any previous astigmatism? The tilt of the figures reminds me of the tilt and elongation of the figures in the paintings of El Greco, the artist, who is believed to have been handicapped by severe astigmatism.

DR. H. DOUGLAS SINGER: In answer to Dr. Gregg, I cannot say whether the patient had astigmatism or not. He never wore glasses.

I do not agree that this is a case of agnosia. In my opinion it is one of apraxia. General apraxia was marked when I first saw the patient but that rather rapidly disappeared under training. He probably did have finger agnosia at the start; he said that it was two or three months before he was able to know which fingers he was dealing with, or what he was doing with them; but at the time we studied him, there was no evidence of finger agnosia. The patient also had a complete agraphia at first. He learned to write, however, but there are still many disturbances in writing, which are reported in the paper.